KANT – QUESTIONS FOR DISCUSSION

Paper under discussion: Friedman, Michael. 2012, 'Kant on Geometry and Spatial Intuition'. *Synthese* Vol. 186: pp. 231–255

1. According to Kant, what is the difference between philosophical cognition and mathematical cognition? (p.232) Do you find this distinction reasonable? (p. 233)

2. The central concept in Kant's philosophy of mathematics is 'construction'. Which work does the concept do in his theory and what exactly does he mean by the term? How does it fit into his overall view of the possibility of (metaphysical) knowledge? (p. 233)

3. In the article, Friedman makes repeated reference to Lisa Shabel's interpretation of Kant's notion of geometrical schemata and construction, and distances himself from her interpretation. In what respect does his view differ from hers? (p. 237, footnote 7)

4. What is Friedman's first objection against a diagrammatic interpretation of Kant's philosophy of mathematics? (p. 239)

5. How do Newton's and Kant's conceptions of space differ? (p. 255)

6. What role, according to Kant, does pure geometry play in our perception of empirical objects?

7. Friedman's goal in the paper is to reconcile the 'logical' (advocated by Jaako Hintikka) with the 'phenomenological' (advocated by Charles Parsons) interpretations of Kant's philosophy of geometry. What is the basic difference between these two interpretations and what is Friedman's strategy for reconciliation? (p. 242, footnote 17)

8. What is the historical meaning and relevance of Kant's philosophiy of mathematics vis à vis the Ancient Greeks and vi à vis his contemporaries? (p. 250)

9. Which influence did Newton's and Leibniz's theories of the geometrization of space have on Kant's theory of geometry? (p. 250)

10. Why, according to Friedman, are Mander's and Shabel's interpretations of Kant inadequate? (p. 254)

FREGE – QUESTIONS FOR DISCUSSION

Frege, Gottlob. 1884. 'The Concept of Number' (Excerpts from *The Foundations of Arithmetic*). Reprinted in: Benacerraf, Paul, and Putnam, Hilary (eds.). 1983. *Philosophy of Mathematics: Selected Readings.* Cambridge: Cambridge University Press: pp. 130-159.

1. How do you understand Frege's claim that a statement of number is an assertion about a concept?
2. What does Frege mean when he says that we cannot know whether Julius Caesar is a number or not?
3. Why, according to Frege, is it impossible to represent to ourselves individual numbers?
4. What solution to unimaginable thought-contents does Frege propose?
5. Why does Frege propose to treat numbers as 'separate' or 'independent'?
6. Frege makes a case for abstract objects. What do you think about that? Do you agree with him? If not, what other way is there to make sense of abstract concepts?
7. Which process does Frege describe to generate a concept of number?
8. What are the problems Frege sees with using one-to-one correspondence as a criterion for equality?
9. Why does Frege talk so much about 'direction' if what he actually wants to find is a definition for 'number'?
10. How do you understand Frege's notion of 'extension of a concept'?
11. How does Frege move from his definition of equinumerosity to his definition of number?
12. Frege defines '0' as "the number which applies to the concept 'unequal to itself'" (p. 145). Could he have just as well defined '0' in terms of 'wooden iron' or 'square circle'?
13. Which difference does Frege identify between his definition of 'following in a series' and Cantor's definition of 'following in a sequence'?
14. What is Frege's criticism of Cantor's definitions of 'following in a sequence' and of 'number'? Do you think that criticism is justified?
15. How do you understand Frege's differentiation between 'laws of nature' and 'laws of laws of nature'?
16. What does Frege criticize about Kant's division of judgements into analytic and synthetic ones?

RUSSELL – QUESTIONS FOR DISCUSSION

Russell, Betrand. 1919. 'Selections from *Introduction to Mathematical Philosophy*'. Reprinted in: Benacerraf, Paul, and Putnam, Hilary (eds.). 1983. *Philosophy of Mathematics: Selected Readings.* Cambridge: Cambridge University Press: pp. 160-182.

1. According to Russell, what is the difference between mathematics and the philosophy of mathematics?
2. Do you agree with Russell's claim that definitions must always begin with undefined terms? Can you think of different ways of defining a set of concepts?
3. What are the issues Russell identifies regarding Peano's axioms?
4. Is there a connection between Frege's Julius Caesar problem and Russell's Cleopatra problem?
5. What does Russell mean when he talks about concepts so simple that they cannot be explained in terms of other, simpler concepts? Does it mean that some concepts are indefinable?
6. Why does Russell think that leaving the three basic terms '0', 'number', and 'successor' undefined is not an adequate basis for arithmetic?
7. Russell points out a subtle difference between 'number' and 'plurality'. What it is and why is it important to be aware of the difference?
8. Russell makes a distinction between definitions 'by extension' and definitions 'by intension'. What is the difference? Do you think the distinction is convincing?
9. "Brown, Jones, and Robinson all of them possess a certain property which is possessed by nothing else in the whole universe, namely, the property of being either Brown or Jones or Robinson."
10. Why is it easier to state whether or not two classes are equinumerous than defining what number is?
11. How does Russell define 'one-one-relation' and what is important about the way he defines it?
12. How does Russell define 'number of a class'? What is the central concept in his definition of 'number'? What is the clever move of his definition?
13. What is odd about his definition of numbers?
14. Frege sees logic as prior to mathematics – what is Russell's view?
15. What is the connection between logical constants and logical propositions?

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HILBERT – QUESTIONS FOR DISCUSSION

Hilbert, David. 1926. 'On the Infinite'. Reprinted in: Benacerraf, Paul, and Putnam, Hilary (eds.). 1983. *Philosophy of Mathematics: Selected Readings.* Cambridge: Cambridge University Press: pp. 183-201.

1. What was Weierstrass' seminal contribution to mathematical analysis?
2. Why, according to Hilbert, are there still disputes about the foundations of analysis?
3. What is Hilbert's central claim regarding the infinite? Does he give an explanation for his claim? What is his ultimate goal?
4. Hilbert asserts that "success is the supreme court to whose decisions everyone submits" (p. 184), in mathematics and everywhere else – what, in your view, constitutes success in mathematics?
5. Hilbert claims to have "established" the finitude of the physical universe, i.e. of both the infinitely small and the infinitely large (p. 186). What is his argumentation and are you convinced by it?
6. He considers the possibility that the infinite is an indispensable concept for human thought and investigates this possibility on the background of the example of mathematics. Where in mathematics, according to Hilbert, does infinity play a crucial role?
7. How did Cantor help make sense of the nature of the infinite? What is his great contribution to mathematical thinking?
8. What, according to Hilbert, is the status of the infinite today (i.e. at 1925), in light of the paradoxes of infinity?
9. Which preconditions of logical thought does Hilbert identify?
10. What, according to Hilbert, are numbers?
11. What role do symbols play in Hilbert's theory of mathematics?
12. Which role does intuition play in Hilbert's account of mathematical symbols?
13. What is the problem Hilbert describes with regard to existence claims about finite vs. infinite totalities?
14. What does Hilbert mean when he says that, from a finitary point of view, a statement like 'a+1 = 1+a' (with 'a' being a numerical symbol) is incapable of negation? What is the problem he sees with that?
15. Hilbert makes a very bold claim: he states that the laws of logic as taught by Aristotle and as used until today do not hold. Why does he say that and what conclusion does he draw from it?
16. According to Hilbert, mathematical symbols are meaningless. Still, we can derive meaningful formulas from them. How does that work?
17. What theory of mathematical proofs does Hilbert propose? What does it achieve?

CARNAP – QUESTIONS FOR DISCUSSION

Carnap, Rudolf. 1956. 'Empiricism, Semantics, and Ontology'. AND 'The Logicist Foundations of Mathematics'. Reprinted in: Benacerraf, Paul, and Putnam, Hilary (eds.). 1983. *Philosophy of Mathematics: Selected Readings.* Cambridge: Cambridge University Press: pp. 241-257/ 41-52.

1. Concerning the possibility of purely empiricist (formalist) interpretation, Carnap makes a distinction between mathematics and physics – is this distinction viable?

2. What is Carnap's overall argumentative goal?

3. What does Carnap mean by 'linguistic framework'? What work does it do?

4. Which two kinds of existence questions can be raised with reference to a linguistic framework?

5. Which factors are relevant for the decision of whether to accept or to reject a language?

6. What is the meaning of mathematical language and which possible answers can be given to the question: do numbers exist?

7. How do you understand Carnap's claim that external questions about the ontological status of numbers lack "any cognitive content" (p. 245)?

8. Carnap seems to hold that, in order to establish a linguistic framework, we need nothing but an initial set of rules. What do you think about that?

9. Carnap argues that propositions cannot be mental events. What is his argument and what do you think about it?

10. What role does choice play in Carnap's theory? How free are we in the choices we make regarding systems of language?

11. How much of a pragmatist is Carnap?

12. What does 'accepting an entity' come down to?

13. What is the relation between the introduction of new linguistic frameworks and one's ontological commitments, according to Carnap? What are your intuitions about this relation?

14. What distinguishes the empiricist's view of meaning from a Platonist view of meaning?

15. How could the dispute between nominalists and Platonists about abstract objects be resolved, and why does Carnap see only slim chances of this ever happening?

INTUITIONISM – QUESTIONS FOR DISCUSSION

Posy, Carl. 2005. 'Intuitionism and Philosophy'. In: Stewart Shapiro (ed.). *The Oxford Handbook of Philosophy of Mathematics and Logic*. Oxford: Oxford University Press: pp. 318-355.

1. Which mathematical problem gave rise to Brouwer's intuitionistic theory, and why?
2. Which role does indeterminacy play for Brouwer's theory? What does he mean by "choice sequences" (p. 323)?
3. What does Brouwer mean by "fleeing properties"?
4. Intuitionistic mathematics as suggested by Brouwer differs from classical mathematics: it is sometimes weaker, sometimes more precise, and sometimes inconsistent with classical mathematics. Which examples for these differences does the text mention?
5. Which parallels does Brouwer see between the activities of ordinary phenomenological and mathematical consciousness?
6. What does Brouwer mean by a "two-ity" (p. 330) and how can it be empty of sensory content if it derives from sensory content? What are your views about this form of abstraction?
7. What are the two "acts of intuitionism" (p. 330)?
8. Brouwer sees mathematics as an independent activity of the creating subject's consciousness, devoid of empirical content. How can he explain parallels between mathematics and empirical sciences, i.e. what accounts for the empirical applicability of mathematics, in spite of its being empty of empirical content?
9. What distinguishes Kant's conception of mathematical knowledge from Brouwer's?
10. What, according to Brouwer, is the ontological status of mathematical objects?
11. Which consequences for classical (non-intuitionistic) mathematics, logic, and language does Brouwer draw from his theory?
12. What are Brouwer's objections to Hilbert's formalism?
13. What is the central difference between intuitionistic logic and classical logic?
14. What is Heyting's view on the ontological status of logic? How does it differ from Brouwer's? What do you think about it?
15. What do you think about Heyting's claim that truth in mathematics is equivalent to provability (p. 340)?
16. How does Heyting argue against the law of the excluded middle?
17. What is Dummett's main contribution to the intuitionist field? How does he argue for his point?
18. Why is it problematic for an intuitionist to assume both an assertability criterion of truth and eternally undecidable propositions?
19. How does Posy propose to solve this tension?

SET THEORY – QUESTIONS FOR DISCUSSION

Boolos, George. 1971. 'The Iterative Conception of Set'. Reprinted in: Benacerraf, Paul, and Putnam, Hilary (eds.). 1983. *Philosophy of Mathematics: Selected Readings.* Cambridge: Cambridge University Press: pp. 486-502.

1. What are the two initial characteristics of sets given in the article?
2. What is "naïve set theory"? Why is it problematic? Which conception of sets is proposed as an alternative?
3. How is the iterative conception of set motivated?
4. Boolos gives a rough description of the stages of the iterative hierarchy of sets. What happens at stage 0, 1, 2, etc.?
5. How does the iterative conception of set avoid the paradox of the set of all sets?
6. What are the axioms of Z (Zermelo set theory)?
7. What axiom is added in ZF (Zermelo-Fraenkel set theory) and what axiom is added in ZFC (Zermelo-Fraenkel-Choice set theory)?
8. What is special about the axiom of extensionality?
9. What is special about the axiom of choice?

Some more general questions to think about:

10. Why has set theory become so popular as a foundational theory of mathematics?
11. How does the set-theoretical picture clarify our understanding of mathematics?
12. Are sets easier to understand than numbers?

FICTIONALISM – QUESTIONS FOR DISCUSSION

MacBride, Fraser. 1999. Listening to Fictions: A Study of Fieldian Nominalism. *British Journal for the Philosophy of Science* Vol. 50: pp. 431-455.

1. To which general problem of philosophy of mathematics does fictionalism propose an answer?
2. What is the overall strategy of fictionalism to answer metaphysical worries about mathematical statements?
3. Fictionalists accept only what is concrete. Does such a view eliminate only metaphysical questions about numbers?
4. What is the indispensability argument for mathematics, and how does fictionalism propose to answer it?
5. How can fictional statements facilitate inferences without standing in a metaphysically substantial relation to the concrete realm? Why are some fictions (like mathematics) useful to science and others (like fairy tales) not?
6. What could fictionalism have to say about pure, i.e. unapplied mathematics?
7. How are nominalistic inferences drawn? What are 'bridge laws'? What are representation theorems'?
8. What is the ontological status of bridge laws and representation theorems? is there a way to define them nominalistically?
9. What is the difference, according to Field, between intrinsic and extrinsic explanations, and how does he employ them? (p. 435;
10. Which procedure does Field propose to nominalize mathematized scientific theories?
11. Field relies on spacetime substantivalism for his fictionalist theory. Why does he need to assume it? Which problems does the view create for a nominalist/ fictionalist?
12. What physical implications does Field's theory create?
13. Field wants to show that mathematics is conservative over science. What does that mean, why does he want to show it, and does he succeed?
14. What do Field and Hale/Wright argue about? How does the argument unfold?
15. How are we supposed to understand the concept 'number' on the fictionalist picture?

STRUCTURALISM – QUESTIONS FOR DISCUSSION

Hellman, Geoffrey. 2005. 'Structuralism'. In: Stewart Shapiro (ed.). *The Oxford Handbook of Philosophy of Mathematics and Logic*. Oxford: Oxford University Press: pp. 536-562.

1. What is structuralism and what triggered its development?
2. Hellman raises five questions which any account of structuralism (and indeed, any account of mathematics) should address. What are these five questions?
3. Which four general types of structuralism are there and what are their basic differences?
4. Why is the choice between first-order or higher-order logic in formulating a theory so central?
5. Why is indefinite extensibility problematic for set theoretic structuralism (STS)?
6. What other difficulties does STS face?
7. What is the main difference between STS and sui generis structuralism (SGS)?
8. How does SGS distinguish mathematical structures from non-mathematical ones?
9. Which problem concerning the identity of places in structures does SGS face?
10. Hellman points out a circularity in SGS which seems impossible to get rid off – which circularity is that?
11. What is the main problem of category-theoretic structuralism (CTS)?
12. Hellman interprets Russell as a structuralist predecessor because of his hypothetical suggestion that arithmetic truths are those truths which hold of any progression, not only the natural number progression (p. 553), and develops this idea into his modal structuralism. What, according to Hellman, is the advantage of a modal structuralism? What are the disadvantages?

MATHEMATICAL TRUTH – QUESTIONS FOR DISCUSSION

Benacerraf, Paul. 1973. 'Mathematical Truth'. *The Journal of Philosophy* Vol. 70 No. 19: pp. 661-679.

1. Which two general kinds of accounts of mathematical truth does Benacerraf identify? (p. 663; accounts that treat the semantics of 1 – 2 – 3 the same and accounts that treat 1 – 2 – 3 differently)

2. What are the two conditions for any account of mathematical truth? (p. 666; must fit into an overall account of truth, and into an overall account of knowledge)

3. What is the standard view? (Platonism) What are the (dis-)advantages of the standard/ Platonist/ realist view? (p. 668; can explain overall account of truth through mathematical objects, but not our access to/ knowledge of those objects)

4. What are combinatorial/ conventionalist accounts? What are their (dis-)advantages? (p. 671; can explain our access to/ knowledge of mathematical truths but not their status as truths, i.e. their semantics)

5. Which two concrete examples for such accounts does Benacerraf give, and which one does he lean towards? (p. 673; Gödel vs. Quine, his inclination is towards Gödel but he sees the defects of both accounts clearly and concludes that no account so far has managed to meet both his initial conditions)

6. Which aspects of mathematics did this course clarify for you? Which aspects remain obscure? What, in your opinion, is the most pressing task for current philosophers of mathematics?

HOLISM – QUESTIONS FOR DISCUSSION

Shapiro, Stewart. 2011. 'Epistemology of Mathematics: What are the questions? What count as answers?'. The Philosophical Quarterly Vol. 61 No. 242: pp. 130-150.

1. Which two sets of issues regarding the epistemology of mathematics does Shapiro identify?
2. Does he manage to explain how we access small finite structures?
3. What are the three steps of pattern recognition, according to Shapiro?
4. What does he mean by his claim that coherence (of structures) counts as evidence for existence? Is the coherence of fictional stories also evidence for their existence?
5. How does Shapiro justify the inductive move from small to large or infinite structures?
6. Does entitlement extend to ontology?
7. What is the difference between consistence, coherence, and satisfiability, and why does Shapiro choose coherence over the other two in order to explain his structuralist account?
8. Consider the role of set theory as the background ontology for structuralism (satisfiability questions) – does the role set theory plays here lead to circularity or not?
9. Does the question whether someone is a realist or an antirealist about mathematical objects have a bearing on the entitlement account?